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Integral optimizing functional of separation efficiency

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Abstract

According to thermodynamics, the integral optimizing functional of separation efficiency (ΔS_S), which is the mixed entropy change between initial and final states directly associated with separation efficiency, was presented to indicate integral separation efficiency for any zone separation method by further considering the entropy change from the uniform to the arbitrary distribution of solute zones. Physically, as a system property, ΔS_S is equal to the amount of information that the solute system obtains from its separation surrounding, or separation system. It can be quantitatively related to the irreversibility of separation processes in the entropy balance equation, and corresponds to the extent of Boltzmann order. In separation science, ΔS_S corresponds to the quantity of separated solutes. For any arbitrary distribution of solute zones, ΔS_S can be calculated directly from the distributions and relative positions of solute zones and the number of moles of solutes. Thus ΔS_S can be calculated directly from the separation results of chromatography and electrophoresis to indicate separation efficiency integrally and quantitatively. For example, for Gaussian distributions of zones, ΔS_S can be calculated directly from the standard deviations of the peaks (σ), the distance between the centers of gravity of adjoining zones (Δx) and the number of moles of the solutes (n) or the peak height (h). A quasi-inverse relation between ΔS_S and separation pureness of solutes (φ) was found numerically. In any effective separation process, ΔS_S is always a negative value, and the more negative ΔS_S is, the better the separation efficiency is. The computer simulation supported the above characters of ΔS_S. The discovery of ΔS_S is a part of the nonequilibrium thermodynamic separation theory, which can be used to integrally optimize and time-varying control the complete separation systems – the aggregates of solute systems and separation systems.

Keywords: Separation efficiency; Integral optimizing functional; Mixed entropy change

1. Introduction

The precise and quantitative expression of separation efficiency of chromatography and electrophoresis and how to achieve optimal separation results by optimizing various operation parameters have always been key issues in separation science. Nonequilibrium thermodynamics provides a formalization for the evolution of the organization of complex systems. Thus, the integral optimizing functional must be established carefully as a part of nonequilibrium thermodynamic separation theory [1–4]. Depending on the integral optimizing functional, the separation processes could be controlled time-varyingly and effectively, and the integral optimization of separations could be determined by computational methods. Moreover, by assessing the impact of fluctuation of various operational parameters in the

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separation processes, the integral optimization of the separation with respect to system fluctuations can be achieved. To realize the integral optimization and to time-varyingly control the robustness separation processes to system fluctuations, the first task is to establish the objective functional, which can not only reflect precisely and integrally the separation efficiency of the solute system, but also can optimize various operation parameters based on the essential physics.

It is necessary to stress that the solute systems consist of two or more components. If only a one-component system is chosen, separation is not principally involved. Separation systems mean all separation surroundings contributing to the separation of solute systems except the solute system itself. We also name the separation system as the environment of the solute system. This is in correspondence with the concepts of the system and environment in thermodynamics. The assemblage of the separation system and the solute system composes the complete separation system, whose various properties and interactions can be described by using certain macroscopic physico-chemical parameters. The essence of the integral optimization is the optimization of the complete separation system. For instance, in capillary electrophoresis, the separation system includes various optimizing parameters, which can be classified into three kinds. They are dynamically adjustable experimental parameters (voltage, cooling temperature, injection length), capillary (whole length, effective length, inside and outside radius, and wall adsorption parameters), and buffer (density, pH, viscosity, electrical and heat conduction). The solute system is mainly described by the parameters that are related to the solute's electrophoretic characteristic, such as the charge, mass and shape of solute [2].

Separation efficiency is not only related to the migration of solutes, but also to column efficiency and various operational parameters, among which there are complex functional relations. Currently it is mainly based on Plate theory that deals with the contribution of the column efficiency and some operation conditions to separation efficiency [5–9]. Moreover, the influence of the contribution of solute relative migrations to separation efficiency is associated with column efficiency by the resolution, R_s [10] or other analogous parameters, such as resolution product and peak capacity [7]. These theories have been guiding and promoting the development of chromatography and electrophoresis in recent several decades and continue to play an important role. However, when understanding separation processes from an alternate perspective, we note that Plate theory is based on the continuity equation of mass conservation focusing on one-component, the main goal of which is to deal with matter diffusion in one-component system. As for this equation itself, its solution of corresponding boundary and initial condition can not give the relations between integral separation efficiency and operation parameters, but just the distribution of one-component. From the fact that Plate theory only deals with one-component, this theoretical system does not involve the separation between different solutes at all. In fact, Plate theory only reflects one aspect of separation (i.e. solute zone broadening), even though the plate height has a profound impact on the separation efficiency. Lately, we also found that the irreversibility of separation processes, (which is caused by thermodynamic forces, and described by the entropy balance equation only in non-equilibrium thermodynamics), is the source of both separating and mixing of solute systems [11]. Thus the optimization of separation efficiency with Plate theory can only be considered as a partial optimization method rather than an integral one. Using R_s or other analogous parameters as the criterion of separation efficiency has visual and intuitive appeal. However, they possess clear deficiencies. For example, with the same R_s, one can obtain different separation efficiencies by changing the relative amounts of the injected components. At this point, R_s appears quantitative, but is essentially a qualitative descriptor, because R_s is not directly related to the quantity of the injection sample [7]. Besides, it is difficult to establish the functional relation between various operation parameters and the integral separation efficiency by using R_s. Because R_s was introduced into separation science only according to the apparent measure, it innately lacks the meaning in physics. What is more important, the most essential property, the irreversibility in separation processes, has not been considered fully.

K. De Clerk and C.E. Cloete adopted specific entropy to indicate the extent of separation [12]. J.C. Giddings described the dissipative (band broadening) roles of entropy, and emphasized that entropy is the single greatest enemy of separations because of its

universal inclination to dilute and remix components that have been so carefully isolated in space [13]. G.H. Stewart researched the entropy measurement and evaluation of separation qualitatively [14]. However, we would find that two bottlenecks limit the development of the entropy concept into separation science fully and quantitatively. First, the net separation entropy change, which is only associated with net separation processes, has not been differentiated from the total mixed entropy change of the solute system in total separation processes. Second, the total mixed entropy change of the solute system is not related to operation parameters with the entropy balance equation. Thus it is just as Giddings thought ''While the formalism of irreversible thermodynamics provides an elegant framework for describing molecular displacements, it provides to little substance and too much conceptual difficulty to justify its development here'' [7].

The functional relationship has been found between the entropy of solute systems and solute distributions with Grand Canonical Ensemble [3]. The kinetic energy distribution of solute molecules is the common essence of the entropy and density distribution of solute zones, and the entropy, entropy change and the rate of entropy change of solute system are a group of characteristic functionals to indicate separation efficiency [1,3]. A nonequilibrium thermodynamic separation model (NTSM) has been proposed in capillary electrophoresis (CE). This model considered the separation processes as the two opposing processing operating in unison: the separating among solute bands and the band broadening. They are in correspondence with the entropy flow and entropy production of solute system in linear non-equilibrium thermodynamics. The functional relations between various operation parameters and separation efficiency were found in CE [2,4].

This paper will consider further the entropy change developed on going from a uniformly mixed sample to an arbitrary distribution of solute zones (e.g. Guassan distribution) to enrich the original optimizing functional [2]. In Gaussian distribution, the integral optimizing functional of the separation efficiency (ΔS_S) was calculated directly from the separation result of chromatography and electrophoresis. The study lays a foundation for integral optimization and the real-time control of the complete separation system with NTSM.

2. Theory

Suppose a solute system is made up of two components i and j with n_i and n_j moles, as shown in Fig. 1. V_0 is the common volume that components i and j occupy in the initial state. V_1, V_{1i} and V_{1j} denote the total volume, the individual volumes that two components i and j occupy in the final state, respectively. In thermodynamics, the total mixed entropy change of solute systems (ΔS) is a state function, which is only related to the final and the initial states, and not to the path of the separation process. Therefore, it is feasible to divide the whole separation process into the following successive processes,

(1) The process of net volume change of the solute system from V_0 to V_1, in which no separation of solutes is involved, corresponds to the mixed entropy change ΔS_1. This represents the entropy associated with sample dilution incurred during the separation processes.

(2) The process of net separation of the solute system, in which the total volume keeps constant while individual volume of each solute changes from V_1 to V_{1i} or V_{1j}, corresponds to the mixed entropy change ΔS_2. This represents the entropy change associated with segregation of the sample components into discrete spacial elements during the separation.

(3) The process of net distribution change from the uniform distribution to Gaussian distribution or others, in which the volume of each solute zone and the total volume of the solute system keep constant, corresponds to the mixed entropy change ΔS_3. This represents a correction to the entropy term ΔS_2 associated with non-uniform distribution of the solutes in volume elements V_{1i} and V_{1j}. In this paper, the non-uniform distribution assumed is Gaussian.

Thus the total mixed entropy change of solute systems in whole separation processes, ΔS, can be divided into three parts, ΔS_1, ΔS_2 and ΔS_3,

$$\Delta S = \Delta S_1 + \Delta S_2 + \Delta S_3 \quad (1)$$

2.1. The mixed entropy changes of the first and second processes

The mixed entropy changes of the first and second processes in Fig. 1 have been presented elsewhere

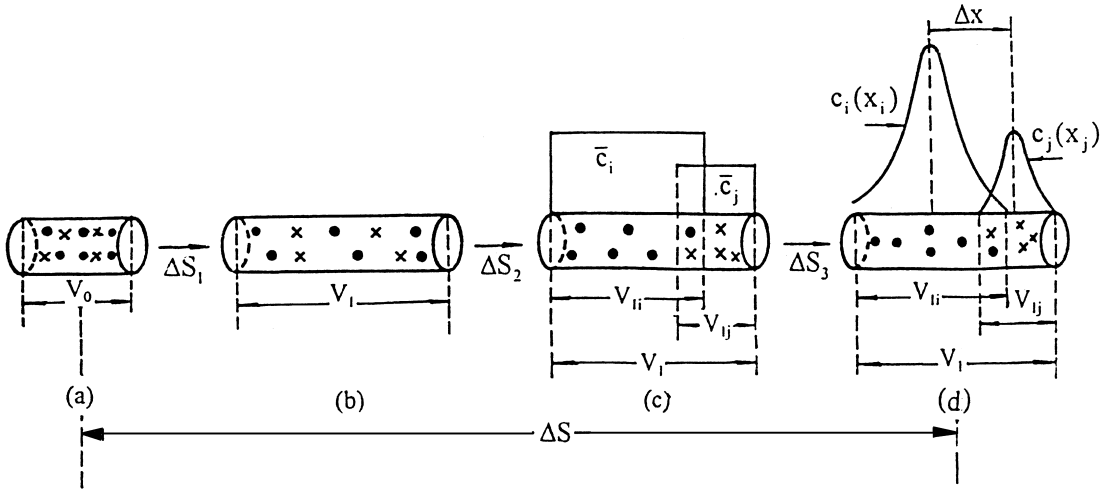

Fig. 1. The spreading and separating of the two-component system in an imaginary separation path. They are respectively the process of net volume changing from (a) to (b), the net separation from (b) to (c), the net distribution changing from (c) to (d). The mixed entropy changes of each process and the whole process are ΔS_1, ΔS_2, ΔS_3 and ΔS, respectively.

[1,2], for the sake of completeness, relevant parts of ΔS are recalled. Obviously, the first process is not concerned with the solute separation directly, but is involved in the total mixed entropy change of the whole separation process. The second and the third processes are concerned with the solute separation. Therefore, the later two processes are the foundations to establish the integral optimizing functional. To concisely describe the relationship between ΔS_S and integral separation efficiency, we assume that separations occur under the conditions of fixed temperature and pressure of the environment of the solute system. According to the results of preceding research [1,2], ΔS_1 and ΔS_2 can be expressed as:

$$\Delta S_1 = (n_i + n_j)R \ln\left(\frac{V_1}{V_0}\right) \tag{2}$$

$$\Delta S_2 = n_i R \ln\left(\frac{V_{1i}}{V_1}\right) + n_j R \ln\left(\frac{V_{1j}}{V_1}\right) \tag{3}$$

where R is the gas constant. If solute zones are Gaussian distribution and their width of datum line is 6σ [7], where σ is the standard deviation of Gaussian peaks. From Fig. 1c,

$$V_1 = 3A[(\sigma_i + \sigma_j) + \Delta x] \tag{4}$$

$$V_{1i} = 6A\sigma_i \quad \text{and} \quad V_{1j} = 6A\sigma_j \tag{5}$$

where A is the cross-sectional area perpendicular to the separation path. Δx is the distance between the centers of gravity of adjoining zones. Inserting Eq. (4) into Eq. (2),

$$\Delta S_1 = (n_i + n_j)R \ln\left\{A\frac{[3(\sigma_i + \sigma_j) + \Delta x]}{V_0}\right\} \tag{6}$$

Inserting Eqs. (4, 5) into Eq. (3),

$$\Delta S_2 = n_i R \ln\left[\frac{6\sigma_i}{3(\sigma_i + \sigma_j) + \Delta x}\right] + n_j R \ln\left[\frac{6\sigma_j}{3(\sigma_i + \sigma_j) + \Delta x}\right] \tag{7}$$

2.2. The mixed entropy change from uniform to Gaussian distribution

The solute zone distribution along the separation path is not uniform in actual separation processes, and therefore is not an equilibrium state, but a nonequilibrium state based on the viewpoint of

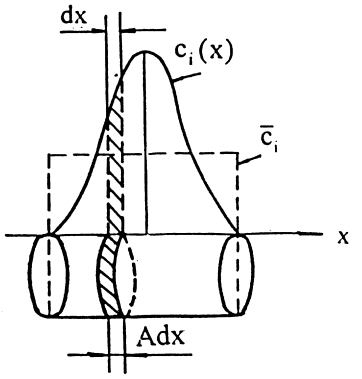

Fig. 2. The distribution of a solute zone. $c_i(x)$ is the actual concentration distribution of component i. $\bar{c}_i$ is the average concentration, and *Adx* is the volume of the cell.

thermodynamics. Thus the separation processes are undoubtedly irreversible processes, and the mixed entropy change is quantitatively related to the irreversibility of separation processes through the entropy balance equation. The entropy in classical thermodynamics is defined in equilibrium states and closed systems, so it is impossible to calculate the mixed entropy change of an open system (where component exchanging occurs among volume cells of solute systems) directly through classical thermodynamics formula. In order to maintain the definition of entropy in equilibrium thermodynamics, we divide the solute zone into many small volume cells, as shown in Fig. 2, which accords with the hypothesis of local equilibrium [15,16]. Suppose each initial volume of cells is *Adx*, as shown in Fig. 3a. In the cell, the distribution of each solute zone is uniform, and the mixed entropy change can be calculated with classical thermodynamics formula directly.

Suppose $c_i(x)$ is the actual concentration distribution of component i in the separation processes, $\bar{c}_i$ is the average concentration of component i in the separation path of 6σ, as shown in Fig. 2. Take any cell as an object, and the mixed entropy change caused by the concentration change of component i from $\bar{c}_i$ to c_i (x) is defined as $d_i s$. The above processes can be divided into two steps in Fig. 3. The first step is the process of dilution (or enrichment), the feature of which is that the quantity of component i in the selected cell is constant while the cell volume changes. So the entropy change (ds_i,b,a) from the state of Fig. 3a to Fig. 3b can be written:

$$ds_i b,a = \bar{c}_i \cdot A \cdot R \cdot \ln\left(\frac{\bar{c}_i}{c_i(x)}\right) \cdot dx \tag{8}$$

where $\bar{c}_i \cdot A \cdot dx = dn_i$, dn_i is the moles of component

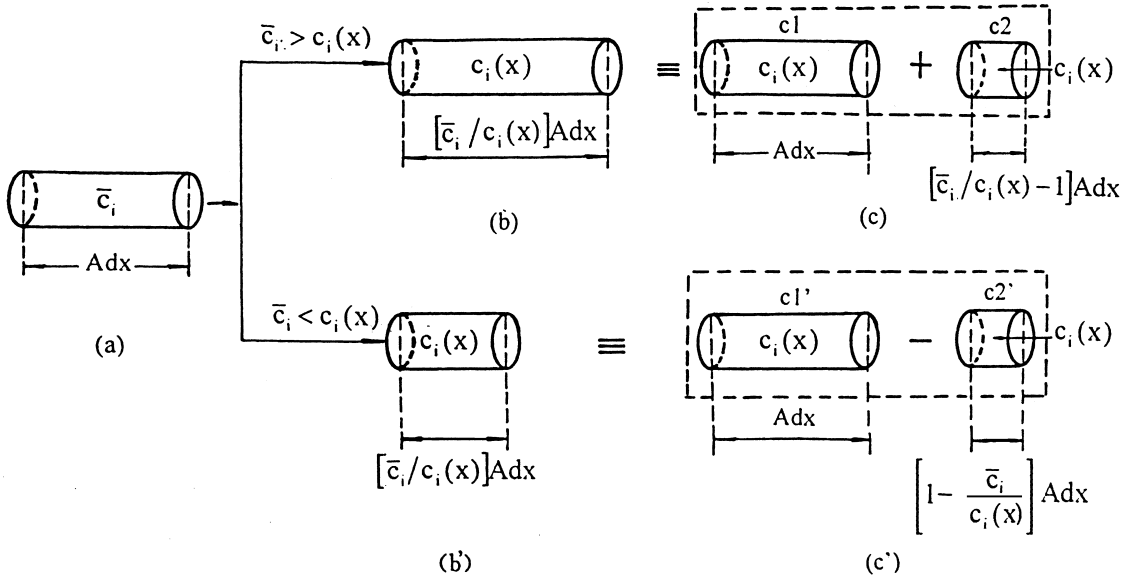

Fig. 3. Schematic drawing is for the calculation of the mixed entropy from $\bar{c}_i$ to $c_i(x)$ in a volume cell (*Adx*).

i in the volume cell. The second step is an invented process from Fig. 3b (or Fig. 3b′) to Fig. 3c (or Fig. 3c′), the feature of which is that the concentration, distribution, quantity and volume of component i are all constant, thus it is an isoentropic process. The Fig. 3c or Fig. 3c′ can be considered as that the volume of the Fig. 3b or Fig. 3b′ is only split into two parts, Adx and $[(\bar{c}_i/c_i(x)) - 1]Adx$. The above process can be expressed

$$s_{i,3b} = s_{i,3c} \tag{9}$$

where $s_{i,3b}$and $s_{i,3c}$ are the entropies of the states in Fig. 3b (or Fig. 3b′) and Fig. 3c (or Fig. 3c′) respectively. Due to the extensive property of the entropy, $s_{i,3c}$ can be written:

$$\begin{aligned} s_{i,3c} &= s_{i,3c1} + s_{i,3c2} \\ &= \tilde{s}_i \cdot c_i(x) \cdot A \cdot dx \\ &+ \tilde{s}_i \cdot c_i(x) \cdot \left[\left(\frac{\bar{c}_i}{c_i(x)} \right) - 1 \right] \cdot A \cdot dx \end{aligned} \tag{10}$$

where $s_{i,3c1}$ and $s_{i,3c2}$ are the entropies of the states in Fig. 3c1 (or Fig. 3 c1′) and in Fig. 3 c2 (or Fig. 3 c2′) respectively. $\tilde{s}_i$ is the partial specific entropy of solute i between the states of Fig. 3b (or Fig. 3b′) and Fig. 3c (or Fig. 3c′). We are only concerned with the mixed entropy change (ds_i) between Fig. 3 c1′ (or Fig. 3 c1′) and Fig. 3b (or Fig. 3b′), thus from Eqs. (8–10), ds_i can be written:

$$\begin{aligned} ds_i &= ds_i,b,a - s_{i,3c2} \\ &= \bar{c}_i \cdot A \cdot R \cdot \ln \left(\frac{\bar{c}_i}{c_i(x)} \right) \cdot dx - \tilde{s}_i \cdot c_i(x) \\ &\cdot \left[\left(\frac{\bar{c}_i}{c_i(x)} \right) - 1 \right] \cdot A \cdot dx \end{aligned} \tag{11}$$

Due to the isoentropic process from Fig. 3b (or Fig. 3b′) to Fig. 3c (or Fig. 3c′), $\tilde{s}_i$ can be expressed as:

$$\tilde{s}_i = \frac{ds_i,b,a}{dn_i} \tag{12}$$

Inserting Eqs. (8) and (12) into Eq. (11),

$$ds_i = -c_i(x) \cdot A \cdot R \cdot \ln \left(\frac{c_i(x)}{\bar{c}_i} \right) \cdot dx \tag{13}$$

Eq. (13) expresses the mixed entropy change from the uniform distribution ($\bar{c}_i$) to the arbitrary distribution $c_i(x)$ of solute i in the volume cell.

2.3. The mixed entropy change of solute system in Gaussian distribution

Suppose w_i is the migration velocity of solute i, and t is the separation time. The entropy change ($\Delta S_{i,3}$) of the whole solute zone from uniform distribution to Gaussian distribution can be written as follows in the integration region of $[w_i t - 3\sigma_i, w_i t + 3\sigma_i]$ with Eq. (13),

$$\begin{aligned} \Delta S_{i,3} &= \int_{w_i t - 3\sigma_i}^{w_i t + 3\sigma_i} d_i s \\ &= -\int_{w_i t - 3\sigma_i}^{w_i t + 3\sigma_i} c_i(x) \cdot A \cdot R \cdot \ln \left(\frac{c_i(x)}{\bar{c}_i} \right) \cdot dx \end{aligned} \tag{14}$$

From the derivation of Eq. (14), $c_i(x_i)$ is not limited to Gaussian distribution. Thus for other distributions, Eq. (14) can be also used.

Suppose $x_i = x - w_i t$ (see Fig. 4), Eq. (14) can be simplified as follows:

$$\Delta S_{i,3} = -A \cdot R \cdot \int_{-3\sigma_i}^{+3\sigma_i} c_i(x_i) \cdot \ln \left(\frac{c_i(x_i)}{\bar{c}_i} \right) \cdot dx_i \tag{15}$$

From Eq. (15), it is clear that $\Delta S_{i,3}$ is not related to Δx, because $\Delta S_{i,3}$ is related only to the distribution

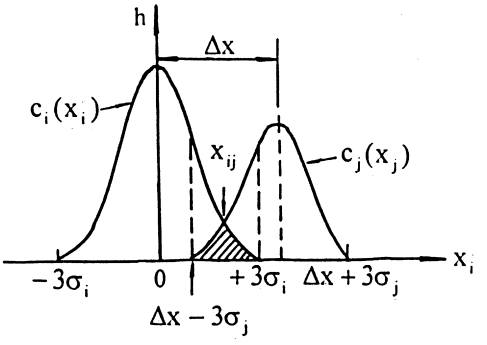

Fig. 4. The distribution of two-component zone. x_{ij} is the intersection point of elution curves of solute i and j.

of one component but not to the relative displacement between its adjacent zones. For the Gaussian distribution, h_i is the peak height or the maximum concentration of solute i. When $x = w_i t$, $h_i = n_i / A\sqrt{2\pi}\cdot\sigma_i$, we have:

$$c_i(x_i) = h_i \exp\left(\frac{-x_i^2}{2\sigma_i^2}\right) \tag{16}$$

According to the extensive property of entropy, from Eqs. (15,16), the mixed entropy change (ΔS_3) caused by the distribution of the solute zone in the two-component system can be written:

$$\begin{aligned}\Delta S_3 &= \sum_{i=1}^{2} \Delta S_{i3} \\ &= A\cdot R\cdot\sum_{i=1}^{2}\int_{-3\sigma_i}^{+3\sigma_i} h_i\cdot\exp\left(\frac{-x_i^2}{2\sigma_i^2}\right)\cdot\left[\frac{x_i^2}{2\sigma_i^2}\right. \\ &\quad \left.+\ln\left(\frac{\sqrt{2\pi}}{6}\right)\right]\cdot dx_i\end{aligned} \tag{17}$$

In deriving Eq. (17), we used the hypothesis, $A\int_{-3\sigma_i}^{+3\sigma_i} c_i(x_i)dx \approx A\int_{-\infty}^{+\infty} c_i(x_i)dx =$ the mole number of injected component i. Inserting Eqs. (6, 7, 17) into Eq. (1), from the separation results (Δx, σ_i and h_i) of solute system in chromatography and electrophoresis, the mixed entropy change (ΔS) of the two-component system during the whole separation process can be obtained,

$$\begin{aligned}\Delta S &= (n_1+n_2)R\ln\{A[3(\sigma_1+\sigma_2)+\Delta x]/V_0\} \\ &\quad + n_1 R\ln[6\sigma_1/(3(\sigma_1+\sigma_2)+\Delta x)] \\ &\quad + n_2 R\ln[6\sigma_2/(3(\sigma_1+\sigma_2)+\Delta x)] + A\cdot R \\ &\quad \cdot\sum_{i=1}^{2}\int_{-3\sigma_i}^{+3\sigma_i} h_i\cdot\exp(-x^2/2\sigma_i^2)\cdot[x^2/2\sigma_i^2 \\ &\quad + \ln(\sqrt{2\pi}/6)]dx_i\end{aligned} \tag{18}$$

2.4. The integral optimizing functional of the separation efficiency

We define a part of the mixed entropy change of the solute system as integral optimizing functional (ΔS_S) in the separation processes, which is directly related to the integral separation efficiency of the complete separation system. Obviously, the entropy change ΔS_2 is related to the separation efficiency. In the actual chromatography and electrophoresis, however, the distribution of the solute zone is generally Gaussian distribution or others. Therefore, to establish the integral optimizing functional, the influence of the actual solute distribution on the separation efficiency should be taken into account. This influence is included in the entropy change ΔS_3, as shown in Fig. 1c and Fig. 1d. Only in this way, the integral optimizing functional can more essentially reflect the actual separation process. Thus it is necessary to carefully analyze the constitution of ΔS_3. In Fig. 4, the distribution of two-component is divided into three parts along the axis x, so ΔS_3 can be expressed

$$\Delta S_3 = \int_{-3\sigma_i}^{\Delta x-3\sigma_j} d_i s + \int_{\Delta x-3\sigma_j}^{+3\sigma_i} (d_i s + d_j s) + \int_{+3\sigma_i}^{\Delta x+3\sigma_j} d_j s \tag{19}$$

In the regions of $[-3\sigma_i,\ \Delta x-3\sigma_j]$ and $[+3\sigma_i,\ \Delta x+3\sigma_j]$ in Fig. 4, there are only separated solutes i and j respectively. The more negative this mixed entropy change in the two regions is, the better the separation efficiency is, and the more separated solutes we have. However, the negative entropy changes in the region of $[\Delta x-3\sigma_j,\ +3\sigma_i]$ do not contribute to the separation of solutes i and j, it can be shown in the second team in the right hand of Eq. (19). All these above can be illustrated in Fig. 4. Thus ΔS_S is composed of ΔS_2 and the first and third teams in the right hand of Eq. (19) with Eqs. (2) and (13),

$$\begin{aligned}\Delta S_S &= n_i R\ln(V_{1i}/V_1) + n_j R\ln(V_{1j}/V_1) \\ &\quad - A\cdot R\cdot\int_{-3\sigma_i}^{\Delta x-3\sigma_j} c_i(x)\cdot\ln(c_i(x)/\bar{c_i})\cdot dx_i \\ &\quad - A\cdot R\cdot\int_{+3\sigma_i}^{\Delta x+3\sigma_j} c_j(x)\cdot\ln(c_j(x)/\bar{c_i})\cdot dx_i\end{aligned} \tag{20}$$

Eq. (20) is the integral optimizing functional of two-component system for arbitrary distribution of solutes. Thus ΔS_S can be calculated directly from the

distributions and relative positions of solute zones and the mole amount of solutes. In other words, we can calculate ΔS_S directly from the separation results (including Gaussian and non-Gaussian distribution) of chromatography and electrophoresis to indicate separation efficiency integrally and quantitatively.

For Gaussian distribution, combined with Eqs. (7), (16), (20), ΔS_S can be written:

$$\begin{aligned}\Delta S_S = {} & \sqrt{2\pi}h_i\sigma_i AR\ln[6\sigma_i/(3(\sigma_i+\sigma_j)+\Delta x)] \\ & + \sqrt{2\pi}h_j\sigma_j AR\ln[6\sigma_j/(3(\sigma_i+\sigma_j)+\Delta x)] \\ & + A\cdot R\cdot\int_{-3\sigma_i}^{\Delta x-3\sigma_j} h_i\cdot\exp(-x^2/2\sigma_i^2)[(x^2/2\sigma_i^2) \\ & + \ln(\sqrt{2\pi}/6)]\cdot dx \\ & + A\cdot R\cdot\int_{3\sigma_i-\Delta x}^{3\sigma_j} h_j\cdot\exp(-x^2/2\sigma_j^2)[(x^2/2\sigma_j^2) \\ & + \ln(\sqrt{2\pi}/6)]\cdot dx \end{aligned} \quad (21)$$

Eq. (21) gave the relation between the integral optimizing functional and the separation results (Δx, σ and h_j) for Gaussian distribution of two-component system. Analogously, the integral optimizing functional of multi-component systems in arbitrary distribution can be also written.

Let $r_h = \dfrac{h_i}{h_j}$ (the ratio of peak heights) and $h_j = 1$, Eq. (21) becomes,

$$\begin{aligned}\Delta S_S = {} & \sqrt{2\pi}\sigma_i r_h AR\ln[6\sigma_i/(3(\sigma_i+\sigma_j)+\Delta x)] \\ & + \sqrt{2\pi}\sigma_j AR\ln[6\sigma_j/(3(\sigma_i+\sigma_j)+\Delta x)] \\ & + A\cdot R\cdot\int_{-3\sigma_i}^{\Delta x-3\sigma_j} r_h\cdot\exp(-x^2/2\sigma_i^2)[(x^2/2\sigma_i^2) \\ & + \ln(\sqrt{2\pi}/6)]\cdot dx \\ & + A\cdot R\cdot\int_{3\sigma_i-\Delta x}^{3\sigma_j} \exp(-x^2/2\sigma_j^2)[(x^2/2\sigma_j^2) \\ & + \ln(\sqrt{2\pi}/6)]\cdot dx \end{aligned} \quad (22)$$

2.5. The separation pureness

Whether in analytical or preparative separations, the separation results are always expressed as the separation pureness of solutes (φ) under the certain collecting mode, which is a basic parameter expressing separation efficiency. The definition of φ is different with the different collection mode. In most cases, the intersection point of solute elution curves is defined as the initial or the final point of solute collecting. For the two−component system, as shown in Fig. 4, the pureness of solute i can be written:

$$\varphi_i = \frac{\displaystyle\int_{-3\sigma_i}^{x_{ij}} h_i\exp\left(\frac{-x^2}{2\sigma_i^2}\right)\cdot dx}{\displaystyle\int_{-3\sigma_i}^{x_{ij}} h_i\exp\left(\frac{-x^2}{2\sigma_i^2}\right)\cdot dx + \int_{\Delta x-3\sigma_j}^{x_{ij}} h_j\exp\left[\frac{-(x-\Delta x)^2}{2\sigma_j^2}\right]\cdot dx} \quad (23)$$

where x_{ij} is the intersection point of elution curves of solutes i and j.

Let $r_h = h_i/h_j$ and $h_j = 1$, then referring to Fig. 4, Eq. (23) becomes,

$$\varphi_i = \frac{\displaystyle\int_{-3\sigma_i}^{x_{ij}} r_h\exp\left(\frac{-x^2}{2\sigma_i^2}\right)\cdot dx}{\displaystyle\int_{-3\sigma_i}^{x_{ij}} r_h\exp\left(\frac{-x^2}{2\sigma_i^2}\right)\cdot dx + \int_{\Delta x-3\sigma_j}^{x_{ij}} \exp\left[\frac{-(x-\Delta x)^2}{2\sigma_j^2}\right]\cdot dx} \quad (24)$$

and the pureness of solute j can be obtained,

$$\varphi_j = \frac{\displaystyle\int_{x_{ij}}^{\Delta x+3\sigma_j} \exp\left(\frac{-(x-\Delta x)^2}{2\sigma_j^2}\right)\cdot dx}{\displaystyle\int_{+3\sigma_i}^{\Delta x+3\sigma_j} r_h\exp\left(\frac{-x^2}{2\sigma_i^2}\right)\cdot dx + \int_{x_{ij}} \exp\left(\frac{-(x-\Delta x)^2}{2\sigma_j^2}\right)\cdot dx} \quad (25)$$

To verify that ΔS_S would be an integral optimizing functional of solute systems, we need to define the alternative pureness parameters, pureness product (φ_p) and average pureness (φ_a)

$$\varphi_p = \varphi_i\varphi_j \quad (26)$$

$$\varphi_a = \frac{(\varphi_i + \varphi_j)}{2} \quad (27)$$

3. Computer simulation methods

For different complete separation systems, the typical solute elution curves and corresponding relationships of ΔS_S and φ to Δx, σ and h can be drawn by MATALAB for windows (The Mathworks, Version 4.2) under the following conditions.

3.1. The influence of r_h on ΔS_S and φ

The influence of r_h on ΔS_S and φ can be seen through Eqs. (22), (24)–(27) when Δx and σ keep constant. Let r_h change from 1 to 32, and $n_i = 20 \times 10^{-6}$ mol, which corresponds to $h_i = 4.064 \times 10^5$ mol/m^3, and $\sigma_i = 10 \times 10^{-3}$ m, $\sigma_j = 15 \times 10^{-3}$ m, $\Delta x = 50 \times 10^{-3}$ m, $A = 19.63 \times 10^{-10}$ m^2, $R = 8.314$ J/mol K. In Fig. 5, a series of elution curves depict the effect of a changing of r_h on the net elution profile calculated using Eq. (16). Under the conditions, Eqs. (22), (24)–(27) show that the curves of ΔS_S (the unit KJ/K) and various pureness (φ_i, φ_j, φ_p and φ_a) vary with the changing of r_h, which is shown in Fig. 6.

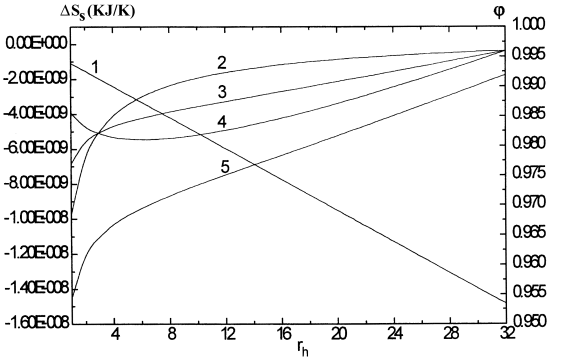

Fig. 6. Under the same conditions of Fig. 5, the curve 1 reflects the relation between ΔS_S and r_h in Eq. (22). Curve 2 between φ_i and r_h in Eq. (24). Curve 3 between φ_a and r_h in Eq. (27). Curve 4 between φ_j and r_h in Eq. (25). Curve 5 between φ_p and r_h in Eq. (26). The vertical coordinate to the left is ΔS_S (KJ/K), and that to the right is percent pureness. The other conditions can be referred to in Section 3.1.

3.2. The influence of Δx on ΔS_S and φ

The influence of Δx on ΔS_S and φ can be seen through Eqs. (21, 23), when n or h and σ keep constant. Δx changes from 0 to 48×10^{-3} m (when $\Delta x > 48 \times 10^{-3}$ m, the two peaks are completely separated). First, let $n_i = \sqrt{2\pi} h_i \sigma_i A$, $n_i = n_j = 20 \times 10^{-6}$ mol, $\sigma_i = \sigma_j = 8 \times 10^{-3}$ m, a series of elution curves depict the effect of a changing of Δx on the elution profile calculated using Eq. (16) in Fig. 7a. In this condition, the curves of Δx on ΔS_S and φ was drawn with Eqs. (21, 23) in Fig. 8a. Second, let $n_i = 20 \times 10^{-6}$ mol, $n_j = 10 \times 10^{-6}$ mol, $\sigma_i = 12 \times 10^{-3}$ m, and $\sigma_j = 8 \times 10^{-3}$ m, with Δx changing from 10×10^{-3} m to 60×10^{-3} m (when $\Delta x > 60 \times 10^{-3}$ m, the two peaks are completely separated), a series of elution curves depict the effect of a changing of Δx on the elution profile calculated

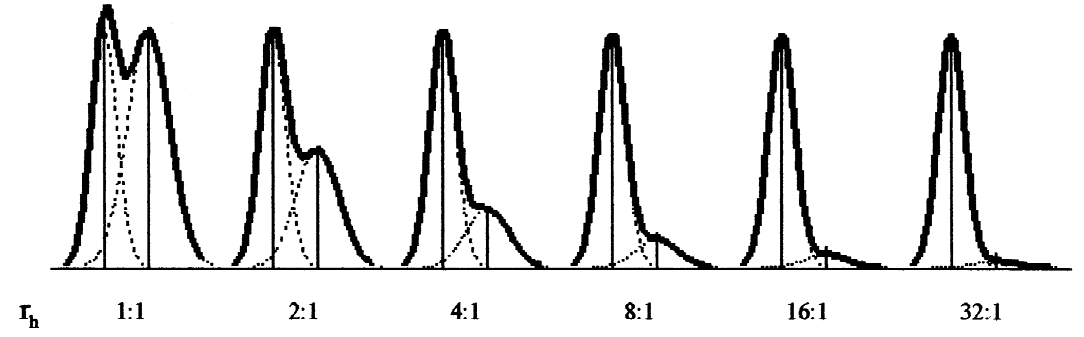

Fig. 5. The elution curves are affected by r_h ($r_h = h_i/h_j$ and $h_j = 1$) under the condition of Δx and σ ($\sigma_j = 1.5\ \sigma_i$) keeping constant with $R_s \equiv 1.0$. The other conditions can be referred to in the Section 3.1.

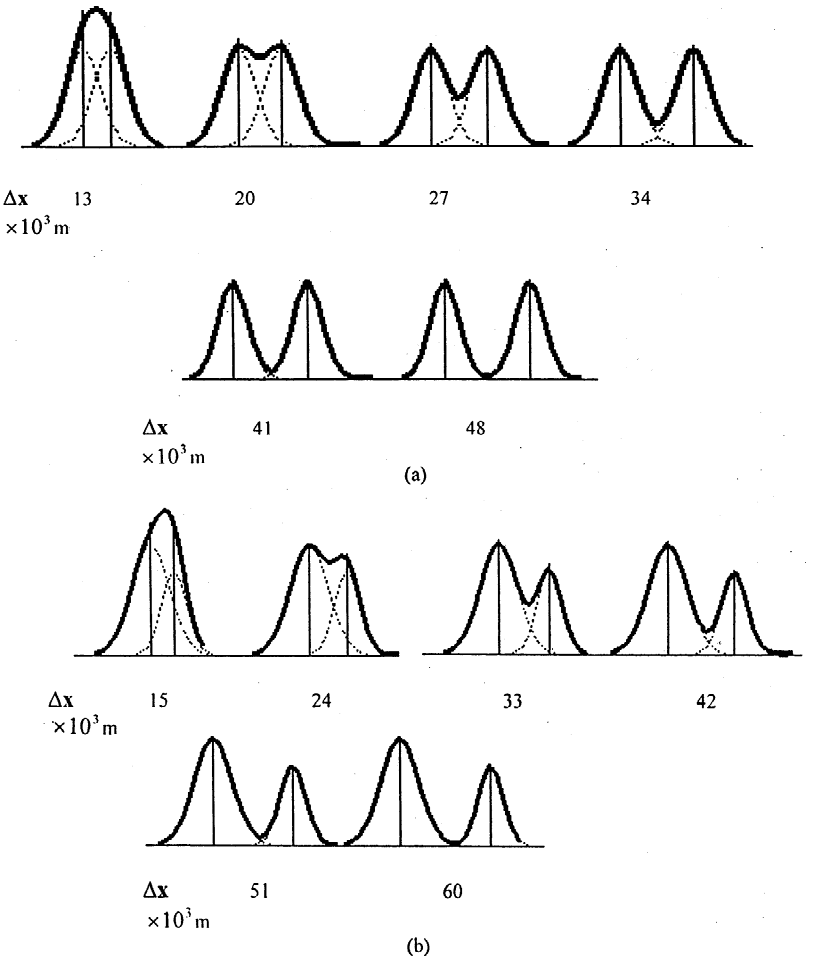

Fig. 7. A group of elution curves with the Δx changing, under the condition of injection samples and σ keeping constant. (a) is for the condition when the σ of the two peeks is equal; (b) is for the condition when the σ of the two peeks is not equal (σ_i:σ_j = 5:3). The other conditions can be referred to in the Section 3.2.

using Eq. (16) in Fig. 7b. Under this condition, the curves of Δx on ΔS_S and φ were drawn by Eqs. (21,23) in Fig. 8b.

3.3. The influence of column efficiency factor r_σ on ΔS_S and φ

The influence of column efficiency factor r_σ (the extent of peak broadening) on ΔS_S and φ can be seen through Eqs. (21,23), when Δx and n or h keep constant. The influences of column efficiency on the similar solutes is basically the same. Thus in this case, it is possible to suppose σ_i and σ_j increase at the same extent, i.e. $\sigma_i = r_\sigma \sigma_{i,0}$, $\sigma_j = r_\sigma \sigma_{j,0}$, where $\sigma_{i,0}$ and $\sigma_{j,0}$ are the basic standard deviations of peaks. Let $\sigma_{i,0} = 2 \times 10^{-3}$ m, $\sigma_{j,0} = 3 \times 10^{-3}$ m, $n_i = 12 \times 10^{-6}$ mol, $n_j = 10 \times 10^{-6}$ mol and $\Delta x = 15 \times 10^{-3}$ m, then a series of elution curves depict the effect of a changing of r_σ on the elution profile calculated using Eq. (16) in Fig. 9. Under this condition, the curves of r_σ on ΔS_S and φ were drawn with Eqs. (21) and (23), as shown in Fig. 10.

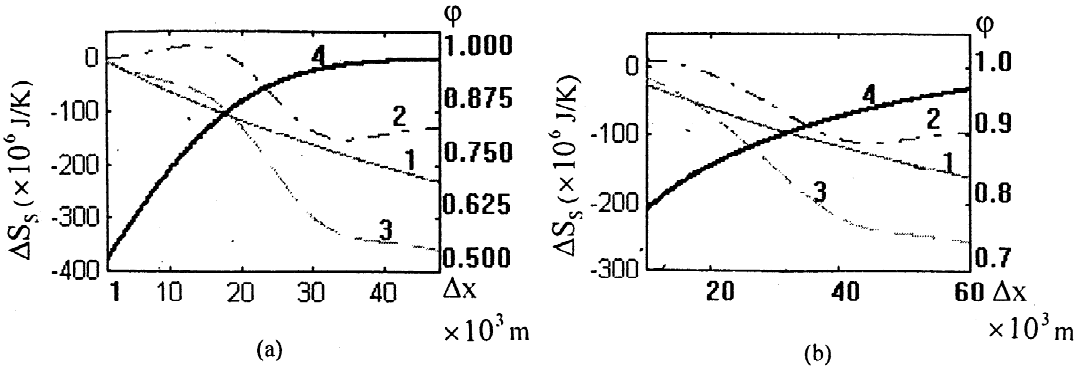

Fig. 8. The influence of Δx on ΔS_S and φ in Eqs. (21,23) under the condition of σ and n keeping constant. In (a), the σ and n of the two zones are equal, and in (b), the σ and n of the two zones are not equal ($\sigma_i:\sigma_j = 5:3$, $n_i:n_j = 2:1$). In both of (a) and (b), the curve 1 is the contribution of the sum of the first and second terms on the right-hand side of Eq. (21) to ΔS_S with Δx changing. Curve 2 is the contribution of the sum of the third and fourth terms on the right-hand side of Eq. (21) to ΔS_S. Curve 3 is the total contribution of all terms in Eq. (21) to ΔS_S. Curve 4 is the influence of Δx on φ in Eq. (23). The other conditions can be referred to in the Section 3.2.

4. Results and discussion

Though ΔS_S is presented as a global functional to indicate separation efficiency, we will only discuss the relationships between ΔS_S and separation results of electrophoresis and chromatography under some special conditions here.

1). If Δx and σ do not change, the resolution (R_s) is surely constant. If the solute zone is a Gaussian distribution, the change of the peak height is in proportion to the quantity of the injection sample. In other words, in Eqs. (21) and (23), only h_i and h_j vary in the same proportion, while others are not changing. Obviously, in this case, the parameter R_s

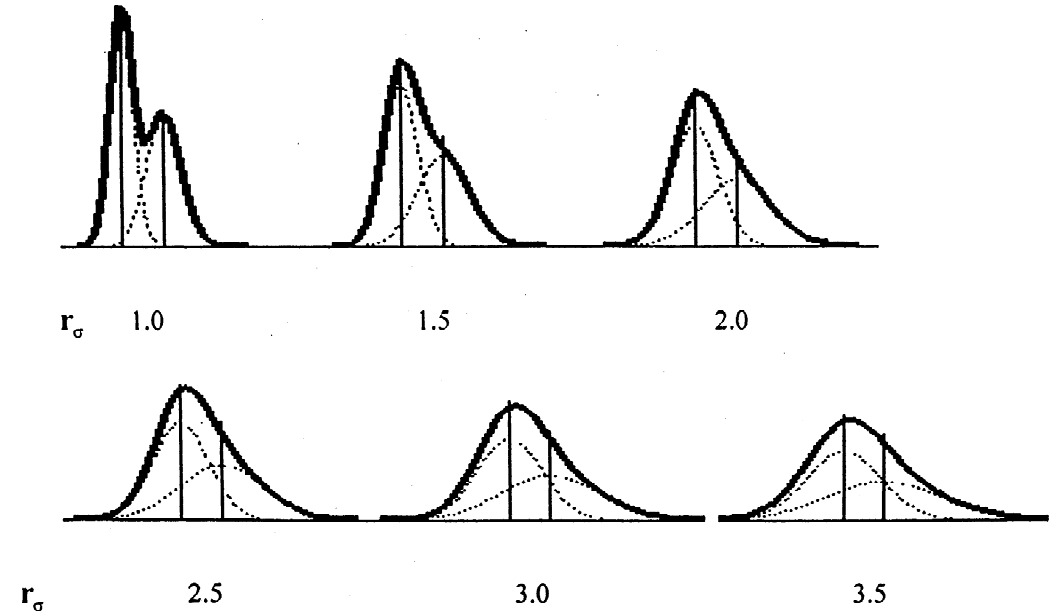

Fig. 9. The elution curves as affected by r_σ under the condition of Δx and n keeping constant. The other conditions can be referred to in the Section 3.3.

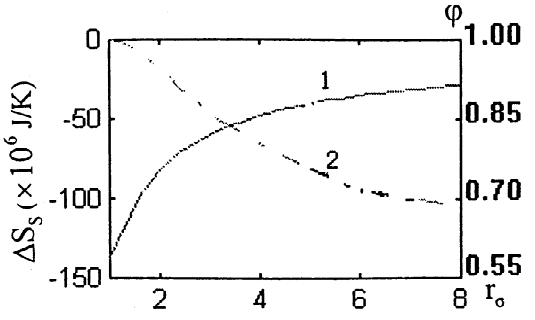

Fig. 10. Curve 1 reflects the relationship between ΔS_S and r_σ in Eq. (21), and curve 2 reflects the relationship between φ and r_σ in Eq. (23), under the condition of Δx and n keeping constant. The vertical coordinate to the left is ΔS_S, and that to the right is φ. The other conditions can be referred to the Section 3.3.

doesn't vary with the quantity of the injection sample. Therefore R_s or the analogy parameters of R_s are not directly related to the quantity of injection sample. But ΔS_S is in proportion to the quantity of injection samples, which can be found clearly in Eq. (21). If the solute proportion of the injection samples is changed, ΔS_S will show the influences of r_h on ΔS_S and various pureness parameters at fixed R_s. Parameter r_h influence on various pureness parameters when R_s keeps constant, which was discussed in many monographs of separation science [7,17]. In fact, this influence is due to the changing of r_h and x_{ij} in Eqs. (23)–(25) under this condition. Of course, it also shows that R_s itself can not reflect the influences of solute quantity. However, Fig. 6 clearly shows that ΔS_S becomes more negative, and φ_p and φ_a become greater as r_h increases. In this case, φ_i and φ_j can only partially indicate the separation efficiency of two-component system. That is to say, as r_h increases, ΔS_S will be more negative, and the quantity of integrally separated solutes (φ_p and φ_a) will become greater, which agrees with the results of computer simulation in Fig. 5. Therefore, when R_s can not express the influence of the quantity and the solute proportion of injection samples on the separation results, ΔS_S can do it.

(2) For the same separation system, the migration velocity of each solute zone may be different, which results in the differences of Δx. In this case, if the column efficiency does not change, the broadening of the solute zones (σ) will keep constant. Under the condition of keeping the quantity of injection samples constant, Fig. 7a and b give out two kinds of elution curves with the same and the different σ, respectively. Fig. 8a and b quantitatively show the tendencies of ΔS_S and φ with Δx changing separately. The contributions of each term in the right-hand side of Eq. (21) to ΔS_S are different with Δx changing. In Fig. 8a, the curve 1 corresponds to the sum of the first and the second terms on the right-hand side of Eq. (21), which expresses the mixed entropy change, resulted from the net separation of solute system. Obviously, the bigger Δx is, the better the separation efficiency is, and the more negative the mixed entropy change is. Curve 2 corresponds to the sum of the third and the fourth terms on the right-hand side of Eq. (21), which expresses the mixed entropy changes resulted from the net distribution changing from uniform to Gaussian, and when the $\Delta x < 20\times10^{-3}$ m, the mixed entropy is a little bigger than zero. It shows that when the centers of gravity of adjoining zones are close, the extent of mixing of two components increases with Δx increasing due to the net distribution of solutes. When 20×10^{-3} m$<\Delta x<40\times10^{-3}$ m, the mixed entropy change becomes more negative with Δx increasing. When $\Delta x > 40\times10^{-3}$ m, the mixed entropy increases a little with Δx increasing. These features of the curve 2 are resulted from the characteristics of Gaussian distribution of solute zone. The process from Fig. 1c to d shows intuitively the features of the curve 2. Curve 3 expresses the integral relationship between ΔS_S and Δx in Eq. (21). In the whole region of Δx changing, ΔS_S decreases continuously in the negative value. Accordingly the separation efficiency becomes better with Δx increasing. Curve 4 expresses that the separation pureness increases with Δx increasing. In addition, Fig. 8a shows the corresponding relationship of ΔS_S and φ.

In Fig. 8b, when Δx is smaller, because of the peak width of solute i being twice as much as that of solute j, the solute j is completely contained by the solute i. The meanings of each curve in Fig. 8b is the same as Fig. 8a. The quantitative relationship reflected by Fig. 8a and b can be demonstrated with elution curves in Fig. 7a and b.

(3) For a fixed the certain solute system, in which solute identities and quantities are keep constant, the separation ability of the complete system varies with operating conditions. The peak height lowers with the broadening of solute zone in Fig. 9. Fig. 10

shows the relationships between ΔS_S, φ and r_σ. The curve 1 expresses that ΔS_S increases with r_σ increasing. Curve 2 expresses that φ reduces with r_σ increasing. The elution curve of Fig. 9 shows that the theoretical prediction of Fig. 10 is reasonable.

We must point out that there are many parameters, e.g. the resolution (R_s), the resolution product, the peak capacity and the chromatographic resolution statistic [18] etc. to indicate separation efficiency, with which most separation scientists are familiar. Introduction of the mixed entropy to indicate separation efficiency may be considered as an excessively complicated device. However, in physics, entropy changes, which can solely quantitatively reveal irreversibility of processes, and correspond to the extent of Boltzmann order. ΔS_S is equal to the amount of information [19–21] that the solute system obtains from its separation surrounding or separation system. The separation state functional between ΔS_S and various operation parameters can be obtained by the entropy balance equation in non-equilibrium thermodynamics. According to separation state functional, we can optimize and control integrally and time-varyingly the various operation parameters to obtain the optimal separation efficiency with the methods of modern cybernetics [21]. Therefore the functional of ΔS_S can not be replaced by other parameters in literature. Otherwise, why did one use the abstruse concept of the entropy instead of those visual parameters to indicate separation efficiency?

Though separation processes are irreversible [21], we have no alternative but to design reversible and idealized processes to calculate the entropy changes of a system between any two states, which is a typical feature of thermodynamic method. In our fully visual field, only solute molecules of the solute system can be found as particles filling the space that they occupy, and the buffer molecules or others are not found in the solute systems at all. Therefore, it is feasible and reasonable to calculate the mixed entropy changes of solute systems between two separation states by taking actually irreversible and non-idealized processes as an idealized equilibrium and reversible processes. In fact, we divided the space that solutes occupy into many small volume cells (see Figs. 2 and 3) and assumed that, although the total solute system is not in equilibrium, there exists within small volume cells a state of 'local equilibrium', which has exceeded equilibrium thermodynamics and has entered non-equilibrium thermodynamics [15].

The state functional ΔS_S is related to separation processes (or separation path of thermodynamics) based on the entropy balance equation in non-equilibrium thermodynamics. And the essence of the entropy balance equation can be expressed [15,23]:

$$S_{gen} = \int_0^t \mathrm{d}t \int^V \sigma_s \mathrm{d}V = \Delta S - S_{flow}$$

$$= S_2 - S_1 - \int_0^t \mathrm{d}t \int^{\Omega} J_{s,tot} \mathrm{d}\Omega \geq 0 \qquad (28)$$

where S_{gen} is the entropy produced inside the solute system in the whole separation processes; t separation time; σ_s the entropy production per unit volume and unit time; V the volume that the solute system occupies; $\Delta S = S_2 - S_1$, the total mixed entropy change of solute systems in the whole separation processes in Eq. (1); S_1 and S_2 are respectively the mixed entropies of solute systems in the initial and final states; $S_{flow} = \int_0^t \mathrm{d}t \int^{\Omega} J_{s,tot} \mathrm{d}\Omega$, S_{flow} the entropy supplied to the solute system by its surroundings in full separation processes; $J_{s,tot}$ the total entropy flow per area and unit time; Ω the boundary of the volume that solute systems occupy.

In Eq. (28), ΔS is separation path independent and is a system property. However, S_{gen} and S_{flow} are separation path dependent, hence, not thermodynamic property. The non-properties, S_{gen} and S_{flow} should never be confused with the thermodynamic property 'entropy change' $(S_2 - S_1)$. As a system property, ΔS_S can indicate any separation efficiency of any separation system between any separation states. In fact, separation scientists can always alter separation paths (S_{gen} and S_{flow}) to obtain different separation efficiencies by adjusting operation parameters in separation processes. Just because ΔS_S is a state functional in thermodynamics, at this point, the discovery of ΔS_S is based on the idea of the black box theory in systematology [22]. To understand the concept of ΔS_S deeply, we have to know the principles of non-equilibrium thermodynamics well [15,23].

On the one hand, for any distributions of solute zones (not only limited to Gaussian distribution), ΔS_S

can be calculated with Eq. (20), if zone's distribution functions were obtained from separation results. In other words, ΔS_S can quantitatively indicate the integral separation efficiency of any electrophoresis and chromatography, and the results of which could be extended to the multi-components. On the other hand, ΔS_S was also related to controllable parameters of complete separation systems with a consistent set of partial differential equations (the conservation of mass and energy, the equation of motion, the entropy balance equation, the phenomenological equations and the Onsager reciprocal relations) in nonequilibrium thermodynamics [2,4,15,16,21]. From Eqs. (1), (13), (20),

$$\Delta S_S = \Delta S_{(\text{Adjustable Parameters})} - \Delta S_1 + A \cdot R \cdot \int_{\Delta x - 3\sigma_j}^{3\sigma_i} [c_i(x) \cdot \ln(c_i(x)/\bar{c_i}) + c_j(x) \cdot \ln(c_j(x)/\bar{c_j})] \cdot dx \quad (29)$$

where $\Delta S_{(\text{Adjustable Parameters})}$ is the total mixed entropy change of solute systems in whole separation processes. It is related to all macroscopic physicochemical parameters, of which many are real-time controllable and optimization parameters. In fact, the introduction of ΔS_S is only an important part of non-equilibrium thermodynamic separation theory [1,2,21].

5. Conclusion

Under certain conditions, the integral optimizing functional, ΔS_S, can be calculated from the separation results of chromatography and electrophoresis. The more negative ΔS_S ($\Delta S_S \leq 0$) is, the better the separation efficiency is. ΔS_S can be used in the non-equilibrium thermodynamic separation theory to optimize and real-time control the complete separation systems.

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